

"Teach A Level Maths"
Vol. 2: A2 Core Modules

11: Proving Trig Identities

Proving Trig Identities

Module C3

AQA

Edexcel

OCR

Module C4

MEI/OCR

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Proving Trig Identities



Any of the trig identities we've met so far, can be used to prove other identities.

So, we need to be familiar with the definitions:

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \cot\theta = \frac{1}{\tan\theta}$$

Also,

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \cot\theta = \frac{\cos\theta}{\sin\theta}$$

and the 3 quadratic trig identities:

$$\cos^2\theta + \sin^2\theta \equiv 1$$

$$\sec^2\theta \equiv 1 + \tan^2\theta$$

$$\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$$

Proving Trig Identities

e.g. 1 Prove that $1 + \tan^2 \theta \equiv \frac{1}{1 - \sin^2 \theta}$

The identity symbol . . . should be used but often isn't.

- The method is to start with the l.h.s. and use any identities to convert it step by step into the r.h.s.
- There is often more than one way of doing it.
- We must always quote the identities used.
- We keep looking at the r.h.s. to see where we want to get to.
- If we get stuck working from the l.h.s., we can start with the r.h.s.

Proving Trig Identities



e.g. Prove that $1 + \tan^2 \theta \equiv \frac{1}{1 - \sin^2 \theta}$

The r.h.s. is a reciprocal containing a square so I want to use an identity for the l.h.s. that involves squares and leads to a reciprocal.

Proof: l.h.s. $\equiv \sec^2 \theta$

Now we need to prove that this is equal to:

We always start like this.

$$\equiv \frac{1}{\cos^2 \theta}$$

Finally, I notice that I have a square of $\cos \theta$ and want a square of $\sin \theta$:

$$\equiv \frac{1}{1 - \sin^2 \theta}$$

$\equiv \text{r.h.s.}$

$$\sec^2 \theta \equiv 1 + \tan^2 \theta$$

$$\sec \theta \equiv \frac{1}{\cos \theta}$$

$$\cos^2 \theta + \sin^2 \theta \equiv 1$$

Proving Trig Identities



Another way of tackling the same problem is as follows:

Prove that $1 + \tan^2 \theta \equiv \frac{1}{1 - \sin^2 \theta}$

Proof: l.h.s. $\equiv 1 + \frac{\sin^2 \theta}{\cos^2 \theta}$

$$\equiv \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}$$
$$\equiv \frac{1}{\cos^2 \theta}$$
$$\equiv \frac{1}{1 - \sin^2 \theta}$$
$$\equiv \text{r.h.s.}$$

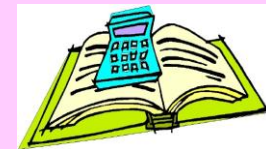
$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$$

(Common denom.)

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

Proving Trig Identities



Exercise

Prove the following identities:

1. $\sec^2 \theta + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta + \tan^2 \theta$

2. $\cos^2 \theta \operatorname{cosec} \theta \equiv \operatorname{cosec} \theta - \sin \theta$

3. $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \equiv 2 \cos^2 \theta - 1$

Proving Trig Identities

Solutions:

1. Prove $\sec^2 \theta + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta + \tan^2 \theta$

Proof:

l.h.s. $\equiv$



Proving Trig Identities

Solutions:

1. Prove $\sec^2 \theta + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta + \tan^2 \theta$

Proof:

l.h.s. $\equiv 1 + \tan^2 \theta +$

$$\sec^2 \theta \equiv 1 + \tan^2 \theta$$

Proving Trig Identities

Solutions:

1. Prove $\sec^2 \theta + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta + \tan^2 \theta$

Proof:

l.h.s. $\equiv 1 + \tan^2 \theta +$

$\operatorname{cosec}^2 \theta - 1$

$$\sec^2 \theta \equiv 1 + \tan^2 \theta$$

$$\begin{aligned} \operatorname{cosec}^2 \theta &\equiv 1 + \cot^2 \theta \\ \Rightarrow \cot^2 \theta &\equiv \operatorname{cosec}^2 \theta - 1 \end{aligned}$$

Proving Trig Identities



Solutions:

1. Prove $\sec^2 \theta + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta + \tan^2 \theta$

Proof:

$$\text{l.h.s.} \equiv 1 + \tan^2 \theta +$$

$$\operatorname{cosec}^2 \theta - 1$$

$$\equiv \tan^2 \theta + \operatorname{cosec}^2 \theta$$

$$\equiv \text{r.h.s.}$$

$$\sec^2 \theta \equiv 1 + \tan^2 \theta$$

$$\operatorname{cosec}^2 \theta \equiv 1 + \cot^2 \theta$$

$$\Rightarrow \cot^2 \theta \equiv \operatorname{cosec}^2 \theta - 1$$

Proving Trig Identities



2. Prove $\cos^2 \theta \operatorname{cosec} \theta = \operatorname{cosec} \theta - \sin \theta$

Proof:

$$\text{l.h.s.} \equiv (1 - \sin^2 \theta) \operatorname{cosec} \theta$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

I noticed that I want $\operatorname{cosec} \theta$ but not $\cos \theta$

$$\equiv \operatorname{cosec} \theta - \operatorname{cosec} \theta \sin^2 \theta$$

(No brackets wanted)

$$\equiv \operatorname{cosec} \theta - \frac{1}{\cancel{\sin \theta}} \times \cancel{\sin}^2 \theta$$

$$\operatorname{cosec} \theta \equiv \frac{1}{\sin \theta}$$

$$\equiv \operatorname{cosec} \theta - \sin \theta$$

(Cancel)

$$\equiv \text{r.h.s.}$$

Proving Trig Identities



$$3. \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \equiv 2 \cos^2 \theta - 1$$

Proof:

$$\text{l.h.s.} \equiv \frac{1 - (\sec^2 \theta - 1)}{\sec^2 \theta}$$

$$\equiv \frac{2 - \sec^2 \theta}{\sec^2 \theta}$$

$$\equiv \frac{2}{\sec^2 \theta} - \frac{\cancel{\sec^2 \theta}^1}{\cancel{\sec^2 \theta}}$$

$$\equiv 2 \cos^2 \theta - 1$$

$$\equiv \text{r.h.s.}$$

$$\sec^2 \theta = 1 + \tan^2 \theta$$

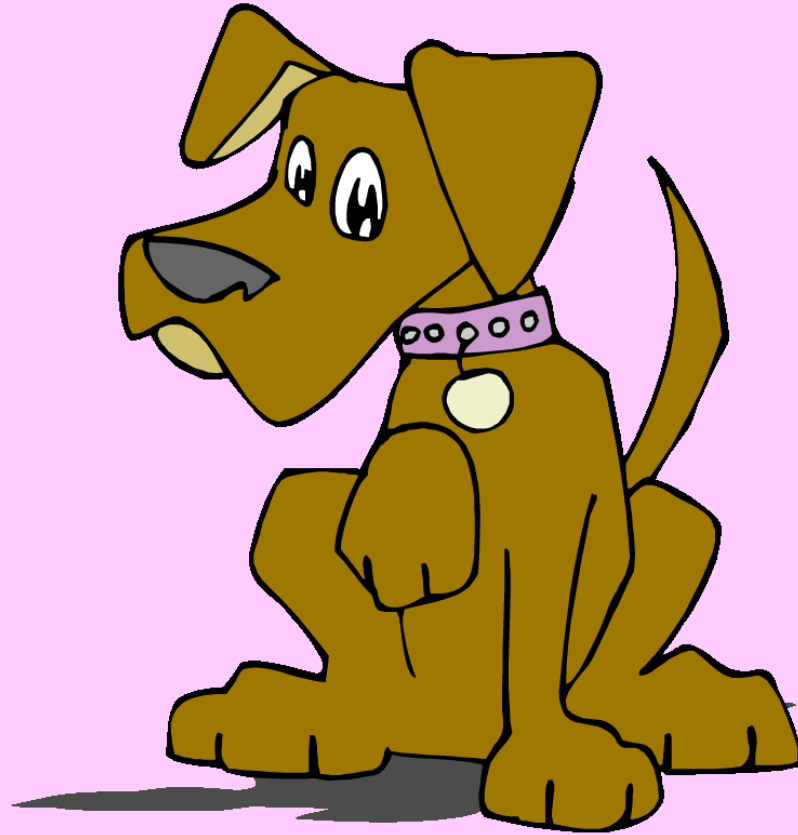
(Used twice)

(Collect terms)

(Split fraction as we want 2 terms)

$$\sec \theta \equiv \frac{1}{\cos \theta} \Rightarrow \cos \theta \equiv \frac{1}{\sec \theta}$$

Proving Trig Identities



Proving Trig Identities

The following slides contain repeats of information on earlier slides, shown without colour, so that they can be printed and photocopied.

For most purposes the slides can be printed as "Handouts" with up to 6 slides per sheet.

Proving Trig Identities

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e.g. Prove that $1 + \tan^2 \theta \equiv \frac{1}{1 - \sin^2 \theta}$

The r.h.s. is a reciprocal containing a square so I want to use an identity for the l.h.s. that involves squares and leads to a reciprocal.

Proof: l.h.s. $\equiv \sec^2 \theta$

Now I change to the reciprocal:

$$\equiv \frac{1}{\cos^2 \theta}$$

Finally, I notice that I have a square of $\cos \theta$ and want a square of $\sin \theta$:

$$\begin{aligned} &\equiv \frac{1}{1 - \sin^2 \theta} \\ &\equiv \text{r.h.s.} \end{aligned}$$

$$\sec^2 \theta \equiv 1 + \tan^2 \theta$$

$$\sec \theta \equiv \frac{1}{\cos \theta}$$

$$\cos^2 \theta + \sin^2 \theta \equiv 1$$