Advanced Math

Chapter 4 Objectives

4.0 Simplifying Complex Numbers

I can use the complex conjugate of a number to simplify

0a) Simplify $\frac{2+3i}{1-2i}$ $\left(\frac{1+2i}{1+2i}\right)$

$\frac{2+7i+6i^2}{1-4i^2+4} = \frac{-4+7i}{4}$

I can simplify complex numbers

0b) Simplify i^{37} $(i^2)^{18} i$

$(1)\, i = i$

4.1 Polynomial Functions

I can determine if an expression is a polynomial

1a) Determine if the following are polynomials:

$3x^2+4x-7$; $4p+p^{-2}$; $\frac{4}{y} = 4y^{-1}$

yes NO NO

I know the definition of a root of a polynomial function

I can state the number of complex roots of a polynomial function

1b) State the number of complex roots of

$6a^4+a^3-2a$ and $3p^2-7p^5-2p^3+5$

4 5

I can determine whether or not a given value is a root of a polynomial

1c) Determine whether 1 is a root of $3x-5=0$

$f(1) = 3-5 = -2$ NO

1d) Determine whether -5 is a root of

$x^3+2x^2-15x=0$ $f(-5) = -125+50+75+0 = 0$

Given roots, I can write the equation of this function in the lowest degree

1e) Write a polynomial equation of least degree with roots $2i, -2i, 3, -3$. yes

$(x-2i)(x+2i)(x-3)(x+3)$

$(x^2+4)(x^2-9)$

x^4-5x^2-36

4.2 Quadratic Equations

I can find the discriminant and determine the type of root based on the value of the discriminant

2a) Find the discriminant of $4x^2-4x-15=0$ and describe the nature of the roots of the equation

b^2-4ac

$16-4(-15)4)$

$16+240$

256 2 REAL

2b) Find the discriminant of $3x^2+2x+5=0$ and describe the nature of the roots of the equation.

$4-4(5)(3)$

-56 2 IMAGINARY

I can solve a quadratic equation by completing the square

2c) Solve $-4x^2 - 11x = 7$ by completing the square

$-4 \quad -4 \quad -4$

$x^2 + \frac{11}{4}x + \left(\frac{11}{8}\right)^2 = -\frac{7}{4} + \frac{121}{64}$

$\left(x + \frac{11}{8}\right)^2 = -\frac{112}{64} + \frac{121}{64}$

$\left(x + \frac{11}{8}\right)^2 = \frac{9}{64}$

$x + \frac{11}{8} = \pm\frac{3}{8}$

$x + \frac{11}{8} = \frac{3}{8}$

$x = -\frac{8}{8} = -1$

$x + \frac{11}{8} = -\frac{3}{8}$

I can solve a quadratic equation by using the quadratic formula

2d) Solve $5x^2 - 14x + 11 = 0$ by the using the quadratic equation

$\frac{14 \pm \sqrt{14^2 - 4(5)(11}}{2(5)}$

$\frac{14 \pm \sqrt{-24}}{10}$

$\frac{14 \pm 2i\sqrt{6}}{10}$

$\frac{7 \pm i\sqrt{6}}{5}$

$x = -\frac{14}{8} = -\frac{7}{4}$

4.3 Remainder and Factor Theorems

I can find the factors of a polynomial equation by using long division

3a) Divide $\left(2x^3 + 3x^2 - 8x + 3\right) \div \left(x + 3\right)$ using long division

$x+3 \;\overline{)\,2x^3 + 3x^2 - 8x + 3}$

$2x^2 - 3x + 1$

$-2x^3 + 6x^2$

$-3x^2 - 8x$

$+3x^2 + 9x$

$x + 3$

$x + 3$

0

I can find the factors of a polynomial equation by using synthetic division

3b) Divide using synthetic division

i. $\left(3x^2 + 4x - 12\right) \div \left(x + 5\right)$

ii. $\left(x^4 - 3x^2 + 12\right) \div \left(x + 1\right)$

1) $-5 \;|\; 3 \quad 4 \quad -12$

$\quad\quad -15 \quad 55$

$\quad 3 \quad -11 \;|\; 43$

2) $-1 \;|\; 1 \quad 0 \quad -3 \quad 12$

$\quad\quad -1 \quad 1 \quad 2$

$\quad 1 \quad -1 \quad -2 \;|\; 14$

I can use the Remainder Theorem to find the remainder and determine whether the binomial is a factor of the polynomial

3c) Find the remainder and state whether the binomial is a factor

$$\left(2x^4+4x^3-x^2+9\right)\div\left(x+1\right)$$

-1 | 2 4 -1 9
-2 -2 3
2 2 -3 | 12 — NO

I can find a missing coefficient of a polynomial given a factor

3d) Find k if $(x-2)$ is a factor of

$$\left(x^4+kx^3-14x^2\right)+0x+0$$

2 | 1 k -14 $0x$ $+0$
2 $4+2k$ $-20+4k$ 0
1 $2+k$ $-10+2k$ $-20+4k$ $-40+8k$ $-40+8k$

$-40+8k=0$
$k=+5$

Given a double root, I can completely factor a polynomial.

3e) Given 2 is a double root of $2x^3-7x^2+4x+4$ factor completely.

2 | 2 -7 4 4
4 -6 -4
2 | 2 -3 -2 | 0
4 $+2$
2 $+1$ | 0

$(x-2)^2(2x+1)$

4.4 Rational Root Theorem

I can list out all possible rational roots

4a) List the possible rational roots of

$$x^3+3x^2-6x-8=0$$

$\pm(1, 2, 4, 8)$

I can use my calculator to guess and check the correct rational roots of a polynomial

4b) Guess and check the rational roots of 4a

$-4, -1, 2$

I can find all the rational roots of a polynomial

4c) Find the rational roots of the polynomial in 4a

$-4, -1, 2$

I can find all real and imaginary roots of a polynomial

4d) Determine all the complex roots of $f(x) = x^3 - 2x^2 - 19x + 20$

Poss RATIONAL $(\pm 1, 2, 4, 5, 10, 20)$

ACTUAL $-4, 1, 5$

4.6 Rational Equations and Partial Equations

I can identify the excluded values of an equation or inequality

I can solve rational equations

6a) Solve $\frac{3x}{2x+1} - \frac{4}{2x-1} = 1$. What are the excluded values? $(2x+1)(2x-1)$

$3x(2x-1) - 4(2x+1) = 4x^2 - 1$

$6x^2 - 3x - 4x - 4 = 4x^2 - 1$

$2x^2 - 7x - 5 = 0$

$(2x-5)(x-1) = 0$

$x = \frac{5}{2} \quad x = 1$

I can decompose a fraction into partial fractions

6b) Decompose $\frac{-3x-29}{x^2-4x-21}$ in partial fractions.

$\frac{-3x-29}{x^2-4x-21} = \frac{A}{(x-7)} + \frac{B}{x+3}$

$-3x-29 = A(x+3) + B(x-7)$

$\frac{-5}{x-7} + \frac{2}{x+3}$

B: $x = 7$, $-50 = 10A$, $-5 = A$

B: $x = -3$, $-20 = -10B$, $2 = B$

4.7 Radical Equations and Inequalities

I can solve radical equations

7a) Solve $\sqrt{x-2} = 6$ and $\sqrt{6x+12} - \sqrt{4x+9} = 1$

$x - 2 = 36$

$x = 38$

$\sqrt{36-2} = 6$

$6 = 6$ ✓

$(\sqrt{6x+12})^2 = (1 + \sqrt{4x+9})^2$

$6x+12 = 1 + 2\sqrt{4x+9} + 4x + 9$

$2x + 2 = 2\sqrt{4x+9}$

$(x+1)^2 = (\sqrt{4x+9})^2$

$x^2 + 2x + 1 = 4x + 9$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0 \quad x = 4 \quad x = -2$

$x = 4$: $\sqrt{36} - \sqrt{25} = 1$ ✓

$x = -2$: $\sqrt{0} - \sqrt{1} = 1$ ✗

I can solve radical inequalities

I can determine extraneous solutions to radical equations and inequalities

7b) Solve $\sqrt{3r+5} > 1$ and $\sqrt{2t-3} < 5$

$(\sqrt{3r+5})^2 = 1^2$

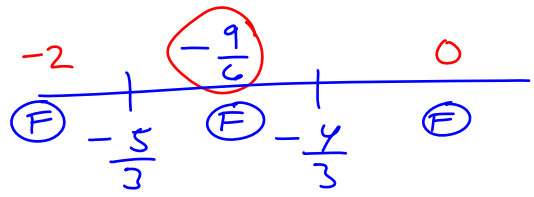
$3r+5 > 1$

$3r > -4$

$r > -\frac{4}{3}$

$3r+5 \geq 0$

$r \geq -\frac{5}{3}$



$x = -2$: $\sqrt{-1} > 1$ F

$x = -\frac{3}{2}$: $\sqrt{-\frac{9}{2}+5} > 1$ F

$x = 0$: $\sqrt{5} > 1$ T

$x \geq -\frac{4}{3}$