Name ________________________________

Period __________

Date: Topic: ***10-3 Composition and Inverses of Functions***	Essential Question: **Does the function $f(x) = x^2$ have an inverse function? Explain your answer.**
Standard: F-BF.1c	Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
***Objective*:**	To find the composite of two given functions and to find the inverse of a given function. Consider the squaring function $f(x) = x^2$ and the doubling function $g(x) = 2x$. As the diagram below shows, these two functions can be combined to produce a new function whose value at *x* is $f(g(x))$, read "*f* of *g* of *x*." *g* is the doubling function *f* is the squaring function $3 \longrightarrow g(3) = 6 \longrightarrow f(g(3)) = f(6) = 36$ $5 \longrightarrow g(5) = 10 \longrightarrow f(g(5)) = f(10) = 100$ $x \longrightarrow g(x) = 2x \longrightarrow f(g(x)) = f(2x) = (2x)^2$ Notice that $f(g(x))$ is evaluated by working from the innermost parentheses to the outside. You begin with *x*, then *g* doubles *x*, and then *f* squares the result. $$f(g(x)) = f(2x) = (2x)^2$$
Summary	

Composition of Functions:	The function whose value at x is $f(g(x))$ is called the ***composite*** of the functions f and g. The operation that combines f and g to produce their composite is called ***composition***.
Example 1:	If $f(x) = 3x - 5$ and $g(x) = \sqrt{x}$, find the following: a. $f\big(g(4)\big)$ $= f\big(\sqrt{4}\big)$ $= f(2)$ $= 3 \cdot 2 - 5$ $= 1$ b. $g\big(f(4)\big)$ $= g(3 \cdot 4 - 5)$ $= g(7)$ $= \sqrt{7}$ c. $f\big(g(x)\big)$ $= f\big(\sqrt{x}\big)$ $= 3 \cdot \sqrt{x} - 5$ d. $g\big(f(x)\big)$ $= g(3 \cdot x - 5)$ $= \sqrt{3 \cdot x - 5}$ Notice in Example 1 that $f(g(x)) \neq g(f(x))$. This is often; although as we shall see, not always the case.

Exercise 1:	If $f(x) = \frac{x}{2}$ and $g(x) = x - 3$, find the following: a. $f\big(g(8)\big)$ b. $f\big(g(-5)\big)$ c. $g\big(f(8)\big)$ d. $g\big(f(-5)\big)$

***Identity Function*:**	The function $I(x) = x$ is called the ***identity function***. It behaves like the multiplicative identity 1. $a \cdot 1 = a$ for all real numbers a $f\big(I(x)\big) = f(x)$ for all functions f For the two functions f and g defined in Example 2, the composites $f(g(x))$ and $g(f(x))$ are both equal to the identity function.
***Example 2*:**	If $f(x) = \frac{x+4}{2}$ and $g(x) = 2x - 4$, find the following: a. $g(1)$ $= 2 \cdot 1 - 4$ $= -2$ b. $f\big(g(1)\big)$ $= f(-2)$ $= \dfrac{-2+4}{2} = \dfrac{2}{2} = 1$ c. $f(-3)$ $= \dfrac{-3+4}{2}$ $= \dfrac{1}{2}$ d. $g\big(f(-3)\big)$ $= g\left(\dfrac{1}{2}\right)$ $= 2 \cdot \dfrac{1}{2} - 4$ $= -3$

	Parts (a) through (d) suggest that the functions f and g "undo each other." Parts (e) and (f) prove that this is so for any number x. e. $f\big(g(x)\big)$ $= f(2x - 4)$ $= \dfrac{(2x-4)+4}{2}$ $= \dfrac{2x+4-4}{2}$ $= \dfrac{2x}{2}$ $= x$ f. $g\big(f(x)\big)$ $= g\left(\dfrac{x+4}{2}\right)$ $= 2\left(\dfrac{x+4}{2}\right) - 4$ $= x + 4 - 4$ $= x$

Exercise 2:	If $f(x) = x^3$ and $g(x) = \sqrt[3]{x}$, find the following: a. $f\big(g(8)\big)$ b. $g\big(f(8)\big)$ c. $f\big(g(-27)\big)$ d. $g\big(f(-27)\big)$

Inverse Functions:

In multiplication, two numbers whose product is the identity 1, such as 2 and 2^{-1} are called inverses. Similarly, two functions whose composite is the identity *I*, such as *f* and g in Example 2, are called ***inverse functions***.

Inverse Functions

The functions *f* and g are ***inverse functions*** if

$$f\big(g(x)\big) = x \text{ for all } x \text{ in the domain of } g$$

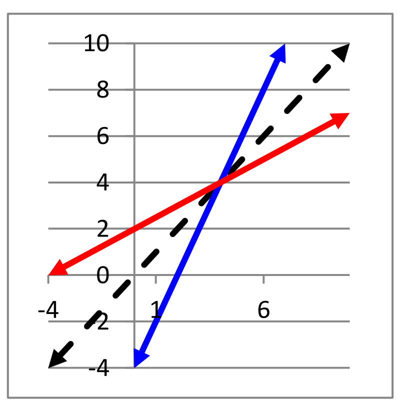
and

$$g\big(f(x)\big) = x \text{ for all } x \text{ in the domain of } f$$

Caution:

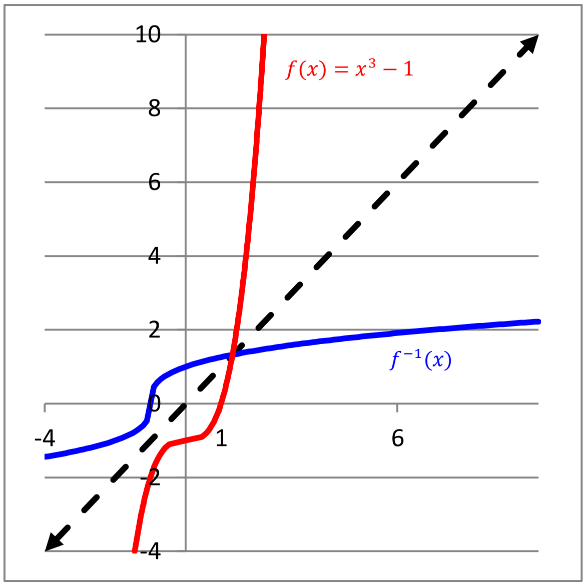
The inverse of a function *f* is usually denoted f^{-1}, read "*f* inverse." The superscript -1 in f^{-1} is ***not*** an exponent. The symbol $f^{-1}(x)$ denotes the value of *f* inverse at *x*; it does ***not*** mean $\frac{1}{f(x)}$.

Suppose that two functions *f* and g are inverses and that $f(a) = b$, so that (*a*, *b*) is on the graph of *f* Then $g(b) = g(f(a)) = a$, and the point (*b*, *a*) must be on the graph of g. This means every point (*a*, *b*) on the graph of *f* corresponds to a point (*b*, *a*) on the graph of g. Therefore, the graphs are ***mirror images*** of each other with respect to the line $y = x$. You can verify this by drawing graphs of inverse functions on the same axes. A computer or a graphing calculator may be helpful. The diagram shows the inverse functions of Example 2.



Horizontal-Line Test:	Some functions do not have inverse functions. If the reflection of the graph of a function *f* is itself to be the graph of a function, the graph of *f* must ***not*** contain two different points with the same *y*-coordinate. Therefore, a function has an inverse function if and only if it is one-to-one. As you learned in Lesson 10-2, every horizontal line intersects the graph of a one-to-one function in at most one point. Therefore, you can use the ***horizontal-line test*** to tell whether a given function has an inverse function. ***Horizontal-Line Test*** A function has an inverse function if and only if every horizontal line intersects the graph of the function in ***at-most*** one point. If a function has an inverse function, you can find it by writing *y* for *f*(*x*), interchanging *x* and *y*, and solving for *y*. Example 3 illustrates.
Example 3:	Let $f(x) = x^3 - 1$. a. Graph *f* and determine whether *f* has an inverse function. If so, graph *f* by reflecting *f* across the line $y = x$. b. Find $f^{-1}(x)$.

Solution:	a. The graph of f is shown in red below. The graph of f passes the horizontal-line test, so f has an inverse. The graph of f^{-1}, shown In blue, is the reflection of the graph of f across the line $y = x$.



b. Replace $f(x)$ by y:

$$y = x^3 - 1$$

Interchange x and y:

$$x = y^3 - 1$$

Solve for y:

$$y^3 = x + 1$$

$$y = \sqrt[3]{x+1}$$

$$f^{-1}(x) = \sqrt[3]{x+1}$$

Exercise 3:	a. Let $f(x) = 2x - 3$. Find $f^{-1}(x)$ b. Let $f(x) = x^3$. Find $f^{-1}(x)$
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